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A Statistical Approach to Mortality Studies

Douglas Brown, ASA, MAAA

Trilogy Actuarial Solutions LLC

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Disclosure

This paper was developed to provide a general overview of the issues related to its subject matter. The comments and recommendations contained in this paper are not intended to provide specific consulting advice or a statement of actuarial opinion. The unique situation of an individual company should always be considered in determining an appropriate response.

Introduction

My approach to mortality studies is undoubtedly colored by my training as a biologist. While I was an undergraduate at the University of Texas, I assisted with a mortality study of radio-collared grizzly bears in Yellowstone park[1]. My job was to code data regarding a bear's date of death from the logs maintained by the park rangers who were tracking and observing the bears by helicopter and to write the C++ code needed to conduct the statistical analysis. Reviewing the rangers' transcripts was a little suspenseful. Locating the collars was easy, but visual sightings were less common. An early sign that a bear may have perished was a stationary collar location over several days. In addition to the death of the bear, this could happen because the collar was dropped or because the bear had not emerged from hibernation yet. Frequently, it could take several days to resolve the bear's status. This frequently involved a ranger locating the collar on the ground.

Mortality studies are frequently conducted by life insurance companies to develop mortality assumptions, validate mortality assumptions, or to compare mortality results between different groups (e.g., testing new underwriting guidelines, comparing results by distributor, or benchmarking against industry studies). All these purposes will benefit from quantifying confidence intervals on the results. For example, suppose that a company has an actual to expected (A/E) ratio of 105% relative to an industry table or that a particular distribution outlet has an A/E ratio 105% relative to the company's expected mortality table. The level of concern and action should depend on whether the desired confidence interval for these results includes 100%.

A statistical approach to evaluating mortality study results (e.g., Monte Carlo simulation) is frequently used in the evaluation of the expected mortality basis in the context of x-factor certifications, but I cannot recall a single instance where I have seen a confidence interval shown in a published actuarial mortality study. I think this is a shame because it seems to me that knowing the width of the confidence interval is very important to understanding the strength of the study's conclusions. The good news is that it should be very easy to incorporate a statistical framework into an existing mortality study. This paper will outline the procedure for incorporating a statistical framework into an existing mortality study. In subsequent papers, I will develop case studies for how this framework can be applied for different applications.

[1] Pease, Craig M., and David J. Mattson. "Demography of the Yellowstone Grizzly Bears." *Ecology*, vol. 80, no. 3, 1999, pp. 957–975. *JSTOR*, www.jstor.org/stable/177030.

Statistical Framework

The statistical framework described by this paper will be added to a mortality study that has been developed by tabulating exposure separately for each policy year during which a subject policy was in force during the study period and calculating the expected number of deaths associated with each exposure interval as the amount of exposure times the expected mortality rate.

The underlying premise of the statistical framework is that you are testing the hypothesis that the expected mortality basis accurately describes the probability of death for the insured lives subject to the study. For this paper, I will use a two-tailed 95% confidence interval for hypothesis testing.

Two interchangeable statistical measures will be developed to determine how well the expected basis fits the observed experience based on the underlying observation that the two-tailed 95% confidence interval for the expected number of deaths is equal to the expected number of deaths ± $1.96 \cdot \sigma$, where σ is the standard deviation of the number of deaths using the expected mortality basis. If a different confidence interval is desired or a one-tailed test is preferred, 1.96 should be substituted with the appropriate value from the standard normal distribution. For example, a two-tailed test with a 90% confidence interval would use 1.64. A one-tailed test with a 95% confidence level would also use 1.64, but only the upper or lower bound would be considered. The calculation of these statistics will depend on whether you are using policy count or face amount as the measure of exposure.

Policy Count

(Actual Deaths – Expected Deaths) / Standard Deviation (σ): This statistic measures the difference between the actual and expected number of deaths in standard deviations. Good fit is indicated using a two-tailed 95% confidence interval when the absolute value of this statistic is less than or equal to 1.96.

Confidence Interval for the A/E Ratio: Good fit is indicated when the when the desired confidence interval includes 100%. The upper and lower bounds of a two-tailed 95% confidence interval are calculated as follows:

Lower Bound: Actual Number of Deaths ÷ (Expected Number of Deaths + $1.96 \cdot \sigma$)

Upper Bound: Actual Number of Deaths ÷ (Expected Number of Deaths – $1.96 \cdot \sigma$)

The standard deviation can be approximated based on aggregated data or calculated for each exposure interval and aggregated (more precise).

Aggregate: $\sigma^2 = \text{exposure} \cdot (1 - q) \cdot q$, where

q is equal to the expected number of deaths divided by the exposure.

Individual Exposure Interval: To calculate the standard deviation more precisely, calculate the variance for each exposure interval as exposure · expected mortality rate · (1 – exposure · expected

mortality rate). The standard deviation at any level of summary is equal to the square root of the sum of the variance.

The statistical analysis assumes that each trial independent. If possible, mortality studies should be conducted by identifying unique lives instead of unique policies. This can pose some logistical challenges, but significant incidence of coverage under multiple policies can bias the study.

Face Amount

Since the statistical framework described in this paper requires that each trial be independent, a precise statistical analysis using face amount will require a technique such as a Monte Carlo simulation. Approximate results can be derived in a traditional mortality study using formulas similar to the policy count case adjusted to reflect the face amount (i.e., replace actual number of deaths with actual claim amount and expected number of deaths with expected claim amount).

To calculate the standard deviation of the claim amount based on the expected mortality basis, calculate the variance for each exposure interval as $(\text{face amount})^2 \cdot \text{exposure} \cdot \text{expected mortality rate} \times (1 - \text{exposure} \cdot \text{expected mortality rate})$. The standard deviation at any level of summary is equal to the square root of the sum of the variance.

My personal preference is to conduct traditional mortality studies on a policy count basis, preferably based on unique lives, to avoid the bias that can occur when the independence assumption required by the statistical analysis is not met. When it is necessary to perform a study on a face amount basis, I use a Monte Carlo simulation whenever possible. In many situations, the amount of insurance can be an important predictive variable. When this is the case, the expected mortality basis should reflect the amount of insurance by making an adjustment or specifying different mortality tables for each relevant face amount band.

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Trilogy Actuarial Solutions is available to assist your organization with mortality studies or establishing mortality assumptions for any purpose (e.g., pricing, asset adequacy testing, principle-based reserves, etc.). You may contact us by phone, email, or through our website. We would be happy to schedule a time to review your situation and collect the information that we need to develop a specific proposal at no expense or obligation to you.

I welcome correspondence regarding this topic. Please feel free to contact me to share your thoughts, opinions, or experiences.

Email: douglas.brown@trilogyactuarial.com

Telephone: (346) 348-2757

Website: www.trilogyactuarial.com

LinkedIn: www.linkedin.com/in/douglas-brown-93725311

